Modelling Prime Diesel Electric Generator Fuel Consumption across Genset Sizings

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***Abstract*—This study investigates a general model for fuel consumption of prime diesel generators within a set range of loadings. The model is parametrized by the rated power output, or sizing, of a generator. This research gives insight into generator fuel consumption characteristics across a wider population of generators to provide a general estimate of fuel consumption for a given sizing. Manufacturer data sheets containing fuel consumption measurements are collected online and through electric power utilities in the Canadian territories for 40 unique diesel generator sizings. The sizing group effects are accounted for through a non-pooled and multilevel regression. Subsequent estimates of linear parameters across generator sizings are modelled through ordinary least squares to obtain the desired model for fuel consumption. The generality and adequacy of this model is investigated through simulation and selected fresh data sources. The general model for prime diesel generator fuel consumption serves as a useful estimate or approximation for subsequent work that requires a general fuel efficiency estimate.**

***Index Terms*—Diesel Electric Power Generators, Remote Power Systems, Microgrid, Regression Modelling**

I. INTRODUCTION

Diesel electric generators play a critical role within the power generation industry. Many emergency or backup power sources are guaranteed through diesel generators. As well, remote communities and industrial operations are often isolated from an interconnected grid, and require a reliable source of electric power often met with diesel generators [1]. Diesel generators are a robust, proven technology, and are capable of providing sufficient adequacy, security, and reliability for a broad range of power generation applications [2].

An essential characteristic of diesel electric power generators is fuel consumption over generator loading, known as the heat rate. Knowing the rate at which generators consume fuel allows for cost projection, modelling, and simulation to be performed. For specific needs, data for a genset will be readily available through public data-sheets or provided by the genset manufacturer or distributor. In addition there exist studies of specific genset fuel efficiency such as [3] or [4]. However, when conducting modelling, simulations, or appropriate sizing of generators, a general estimate of a generators fuel consumption may be more useful. An approach of using data from specific gensets may be lacking. Examples of studies where a more general estimate of generator fuel consumption may have been useful are [5], [6], and [7]. An energy-flow model is developed for performance analysis and sizing for a wind-diesel microgrid in [5]. An optimal sizing and location scheme for remote solar-diesel systems is developed based on economic, environmental, and technical factors to minimize long run costs in [6]. Simulations to investigate multiple operating strategies for wind-diesel and/or solar-diesel small remote systems are investigated in [7]. However only a single heat rate value is assumed to calculate savings in fuel expenses in [7]. Fuel consumption data only from select generators are used in [5]. The slope and intercept of the fuel consumption curve for the modelling in [6] are taken from [4].

The fuel consumption curve is well understood to be generally linear [8] across constant speed generators, and is assumed as such in popular software such as HOMER [9]. However, the relationship between this curve and generator sizing is less understood. Clearly there will be a causal relationship between generator sizing and the fuel consumption curve, however the exact nature of said relationship across a wider population of gensets has not been well studied within the literature.

To this end, specific generator properties within the population of diesel generators had to be selected to ensure a sufficiently applicable model. This study is restricted to prime rated diesel gensets of 60 Hz frequency, which are common in North America.

The goal of this study is to estimate a model for genset fuel consumption as a function of loading, entirely parametrized by the rated power output of a generator. The slope and intercept of the linear fuel consumption curves are estimated across a sample of genset sizings. Functional relationships between the linear parameters and genset sizings are estimated and investigated. A final general model is presented that creates a fuel consumption curve entirely through rated genset power output. The linear fuel consumption curve is given by,

$$F = b_1 + b_2 P \tag{1}$$

where F represents volumetric flow in $\frac{\mathrm{L}}{\mathrm{h}}$, P genset loading in kW, with b_1 and b_2 the usual linear parameters of intercept and slope, in $\frac{\mathrm{L}}{\mathrm{h}}$ and $\frac{\mathrm{L}}{\mathrm{kWh}}$ respectively. The fuel consumption curve (1) can be transformed into a heat rate curve, given in $\frac{\mathrm{kWh}}{\mathrm{L}}$. The heat rate is important as a measure of generator

TABLE I: Considered Data

Genset Sizing [kW]	Number of Data points	Genset Sizing [kW]	Number of Data points
36	7	520	12
45	8	545	31
54	11	600	12
72	7	680	26
90	7	725	15
95	4	818	4
120	4	835	4
135	7	890	12
157.5	7	895	12
180	7	900	8
210	4	920	4
225	7	1045	12
250	7	1100	16
270	4	1135	7
320	18	1245	12
350	4	1286	4
365	20	1350	4
410	23	1365	4
450	4	1450	12
455	22	1600	11

efficiency, and is frequently utilized in assessing diesel electric generator performance. The heat rate transformation is given by,

$$\text{Heat Rate} = \frac{P}{F} = \frac{P}{b_1 + b_2 P} \tag{2}$$

and is a non-linear transformation. The goal of this research to estimate b_1 and b_2 from a given population of diesel generators to fit a general model describing fuel consumption with respect to genset sizing.

II. Data Sources

Generally there are two types of fuel consumption data available for prime diesel generators based on sample size. The first type has fuel consumption measurements available at a larger sample size, measured at a fixed pattern of loadings, typically across 10% and 100% of rated generator output. The second type has a smaller sample size, and has fuel consumption measurements at 50% 75% 100% or 25% 50% 75% 100% of rated power output. These data categories will be referred to as maximal and minimal data categories. Data-sheets with minimal fuel consumption data are readily available from a variety of sources, including manufacturers, distributors, and third-party websites. Data-sheets with maximal fuel consumption data are not readily available.

Select prime generator's data-sheets were made available by ATCO Electric Yukon, Northwest Territories Power Corporation, Yukon Energy Corporation, and Qulliq Energy Corporation that comprise the maximal fuel consumption data for this study. The set of maximal data generators were used as a guide to inform selection of remaining generator data sources. That is, minimal data sources were selected to ensure a sufficient range of data across genset sizings. No minimal data sources were chosen beyond 300 kW above or below the highest and lowest available maximal data genset sizing.

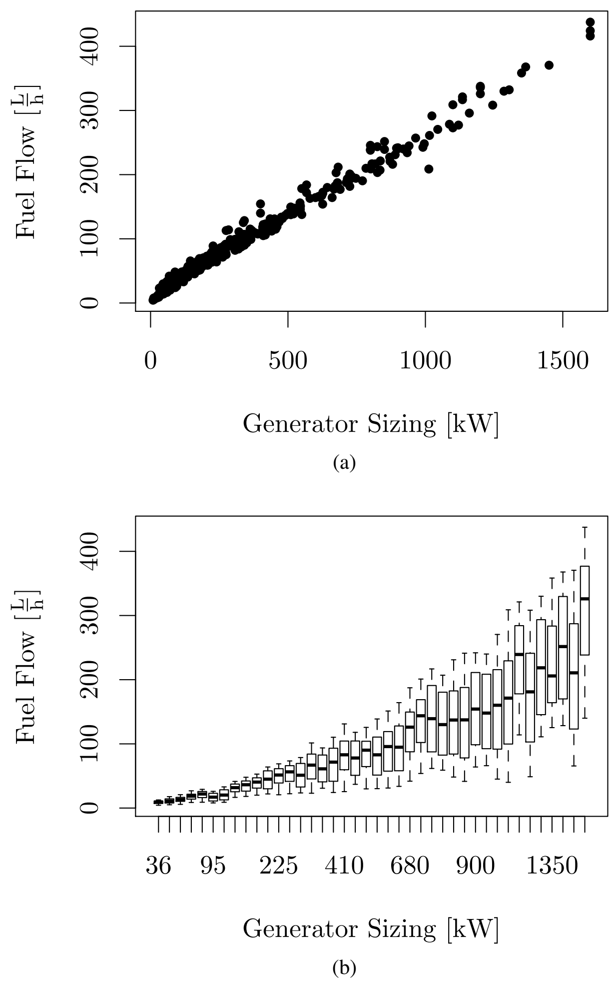


Fig. 1: (a) Pooled fuel consumption data. (b) Boxplot of fuel consumption grouped by generator sizings.

Throughout all datasheets used within this study, fuel consumption data were available only from approximately 10% of rated genset power output to 100%. Note that the linear fuel consumption for all generators surmised earlier is typically insufficient at some indeterminate lower loading threshold, as a genset cannot consume a significantly positive or negative amount of fuel when there is no load. Diesel gensets are also known to experience drops in efficiency when overloaded [10], implying the linearity assumption is insufficient for loadings above 100% of rated output. Thus this study is only concerned with estimating fuel consumption curves for loadings within 10% and 100% of rated output given the lack of data for observations outside this range. There are 40 sizings of gensets collected for this study, representing a total of 404 data points. Table I provides further detail on the genset sizings as well as a sample size at each level.

A plot of the pooled fuel consumption data is given in Fig. 1a. The linearity of the data is immediately apparent, however, consideration must be made of the group effect of generator power rating. That is, the relationship between fuel consumption and genset sizing may have an effect on the overall relationship between fuel consumption and loading in the data. This is investigated in Fig. 1b, wherein boxplots describing fuel consumption across genset sizings are shown. The median fuel consumption increases, which is expected. Additionally, the absolute variability increases substantially. This is also expected, an 800 kW generator will have a larger range of data than a 200 kW generator. However, in addition to the expected behaviour in Fig. 1b there may be less obvious group effects present.

III. Modelling the Group Level Data

A. Linear and multilevel regression approaches

The goal of this study is to estimate a model for genset fuel consumption as a function of loading, entirely parametrized by the rated power output of a generator. A regression on the pooled data in Fig 1a. is the most simplistic approach for assessing how fuel consumption varies with respect to loading,

$$F_m \sim \mathcal{N}(\beta_0 + \beta_1 P_m, \sigma^2), \quad m = 1, \ldots, 404 \tag{3}$$

where F_m denotes the m^{th} fuel consumption in $\frac{\text{L}}{\text{h}}$, $\mathcal{N}$ represents the normal distribution function, β_0 and β_1 are intercept and slope parameters in $\frac{\text{L}}{\text{h}}$ and $\frac{\text{L}}{\text{kWh}}$ respectively, P_m represents the m^{th} loading in kW, and σ^2 is the variance. However this model ignores all group variation in the collected data. That is, changes between fuel consumption across genset sizing are not accounted for since all data are pooled together. Another possibility is a regression on the non-pooled data. This entails fitting a separate model at each group level,

$$F_{km} \sim \mathcal{N}(\beta_{0k} + \beta_{1k} P_{km}, \sigma_k^2), \quad k = 1, \ldots, 40 \tag{4}$$

where F_{km} is the fuel consumption for the m^{th} loading in the k^{th} genset sizing, P_{km} is the load, and so on. This model is superior to the pooled regression in that it accounts for group variation in fuel consumption. A drawback of this approach is it typically places too much emphasis on the group effects. That is, for a particular generator sizing no information from the other 39 sizings are incorporated into the model. Each fuel consumption model with respect to generator sizing is given equal importance regardless of sample size, data quality, or other pertinent factors. This approach may overstate the effect of rated power output on fuel consumption, in other words, overfit.

The pooled and non-pooled regressions exist at opposite ends of a spectrum in model specificity. In contrast, a multilevel model (also known as a hierarchical model or mixed-effects model) exists as a compromise between the two extremes. Following the explanation given in [11], multilevel regression is a generalization of typical linear regression, where the regression coefficients are allowed to vary by group effects in the model rather than being treated as fixed constants. The linear parameters are assigned a probability distribution, wherein parameters of said probability distributions are estimated from the data. In symbols,

$$\begin{gathered} F_m \sim \mathcal{N}(\beta_{0km} + \beta_{1km} P_m, \sigma^2) \\ \begin{pmatrix} \beta_{0k} \\ \beta_{1k} \end{pmatrix} \sim \mathcal{N}\left(\begin{pmatrix} \mu_{\beta_0} \\ \mu_{\beta_1} \end{pmatrix}, \begin{pmatrix} \sigma_{\beta_0}^2 & \rho\sigma_{\beta_0}\sigma_{\beta_1} \\ \rho\sigma_{\beta_0}\sigma_{\beta_1} & \sigma_{\beta_1}^2 \end{pmatrix} \right) \end{gathered} \tag{5}$$

where the linear parameters β_0 and β_1 are assumed to be jointly normally distributed, such that μ_{β_0} and σ_{β_0} are the mean and standard deviation of β_0; μ_{β_1} and σ_{β_1} are the mean and standard deviation of β_1; and ρ is the correlation coefficient between β_0 and β_1. The fitting process of (5) differs from typical linear regression such that in addition to the regression performed on the response and explanatory variables, a regression between the k group variables and the linear parameters β_0 and β_1 is performed. The result of this distinction is that the multilevel regression estimates of β_{0k} and β_{1k} are comprised of information from both the group level effects and the pooled data.

A more compact matrix notation of (5) is used for clarity,

$$\begin{aligned} &\mathbf{y} = \mathbf{X}\boldsymbol{\beta} + \mathbf{Z}\mathbf{b} + \mathbf{e} \\ &\mathbf{b} \sim \mathcal{N}(\mathbf{0}, \boldsymbol{\Sigma}), \quad \mathbf{e} \sim \mathcal{N}(\mathbf{0}, \mathbf{R}) \end{aligned} \tag{6}$$

where $\mathbf{y} = F_m$, $\mathbf{X}$ and $\mathbf{Z}$ are design matrices linking the $\boldsymbol{\beta}$ regression coefficients and the $\mathbf{b}$ random predictors to each observation in the data, with $\mathbf{b}$ and $\mathbf{e}$ representing independent random variables. The matrices $\boldsymbol{\Sigma}$ and $\mathbf{R}$ are covariance structures. The β_{0k} and β_{1k} parameters seen in (5) have been separated into fixed ($\boldsymbol{\beta}$) and random ($\mathbf{b}$) components.

Similar to typical linear regression, a simple multilevel regression approach assumes homogeneity of variance among group level data. However, as shown in Fig. 1b. this assumption is clearly violated by the data, and indeed is expected to be violated. Thus the model in (6) is amended to accommodate the heterogeneity of the group level variance. Following [12], the variance function for (6) is defined as

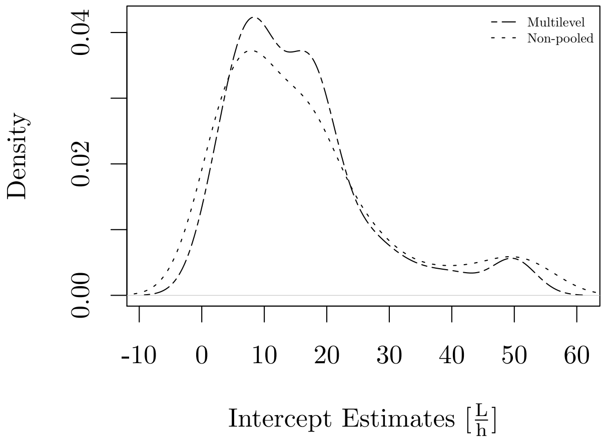
$$\operatorname{Var}(\mathbf{e}|\mathbf{b}) = \sigma^2 g^2(\boldsymbol{\mu}, \mathbf{v}, \boldsymbol{\delta}) \tag{7}$$

where $\boldsymbol{\mu} = \operatorname{E}[\mathbf{y}|\mathbf{b}]$ and E is the expected value operator, $\mathbf{v}$ is a matrix of variance covariates, $\boldsymbol{\delta}$ is a vector of variance parameters, and $g(\cdot)$ is a variance function. Referring to Fig. 1b again, the variance seems to exhibit non-linear growth across rated genset power. Thus the variance function specification will be of the form,

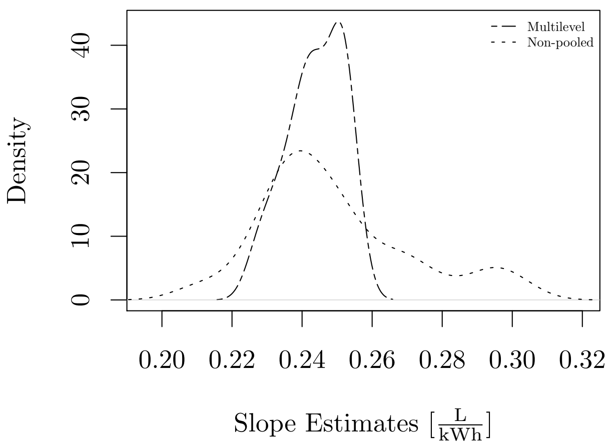
$$\operatorname{Var}(\mathbf{e}) = \sigma^2 |\mathbf{v}|^{2\boldsymbol{\delta}} \tag{8}$$

where $\boldsymbol{\delta}$ are parameters to be estimated. Note that if linear growth was observed $\boldsymbol{\delta}$ would be fixed at $\frac{1}{2}$.

All model fitting is done through R statistical software [13]. Multilevel modelling is performed with the "nlme" package [14]. The "nlme" package fits multilevel models through a maximum likelihood or restricted maximum likelihood procedure. The likelihood approach considers the regression coefficients and variance parameters within the likelihood function, whereas in the restricted approach only the variance components are included in an initial likelihood function, with



(a)



(b)

Fig. 2: (a) Density curves for multilevel and non-pooled regression estimates of intercept. (b) Density curves for multilevel and non-pooled regression estimates of slope.

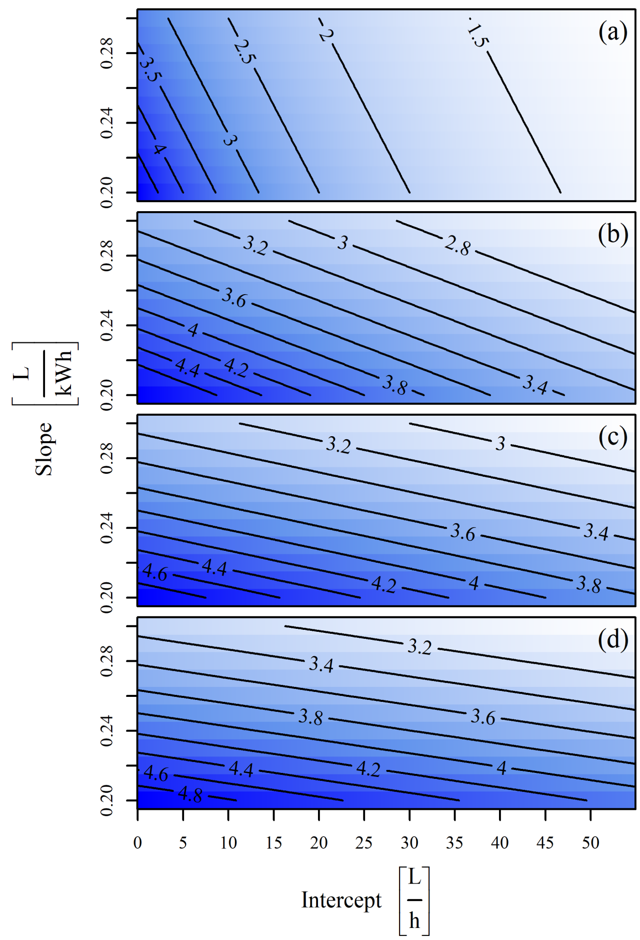


Fig. 3: Contours of the heat rate curve in (2) as a function of estimated slope and intercept range at a (a) 100 kW loading, (b) 500 kW loading, (c) 900 kW loading, and (d) 1300 kW loading.

regression coefficients estimated in a subsequent step [15]. The restricted likelihood approach is known to produce statistically better estimates than the straightforward likelihood approach [16]. Hence the restricted likelihood approach is used.

B. Model assumptions

Typical linear regression and linear multilevel regression share similar assumptions regarding the homogeneity of variances and normality of error. The homogeneity assumption was addressed in (8). The normality assumption is ignored. The goal of this study is not to make predictive inferences upon the wider population of diesel gensets; rather to estimate the patterns found within fuel consumption data across a specific range of genset sizings. Inferences regarding the normality of the multilevel errors are of no concern within the study scope.

C. Investigation of multilevel and non-pooled estimates

Empirical density estimates for the estimated slope and intercepts from the multilevel and non-pooled regression are given in Fig. 2a and Fig. 2b. respectively. The density curve for the multilevel estimates of intercept is marginally tighter than the non-pooled regression estimates, where the density curve for the multilevel estimates of slope is far tighter than the non-pooled estimates. It is clear the non-pooled regression gives a wider range of estimates than the multilevel approach. These figures indicate that the non-pooled model is overfitting the data by giving more importance to each genset sizing, as anticipated. The variability of the non-pooled slope estimates is particularly egregious; with the density curve far wider and flatter. Hence the multilevel model's estimates of slope and intercept are selected to build the general model.

Recall the expression for the heat rate as a function of fuel consumption in (2). Fig. 3 shows (2) as a function of the range of the estimated intercept and slope coefficients. The contours of the heat rate function illustrate how the range of estimated slope and intercept parameters within this study affect said heat rate. At the 100 kW loading the intercept parameter has a greater effect on the heat rate, whereas at the 1300 kW loading, the slope parameter has the greater effect. For the

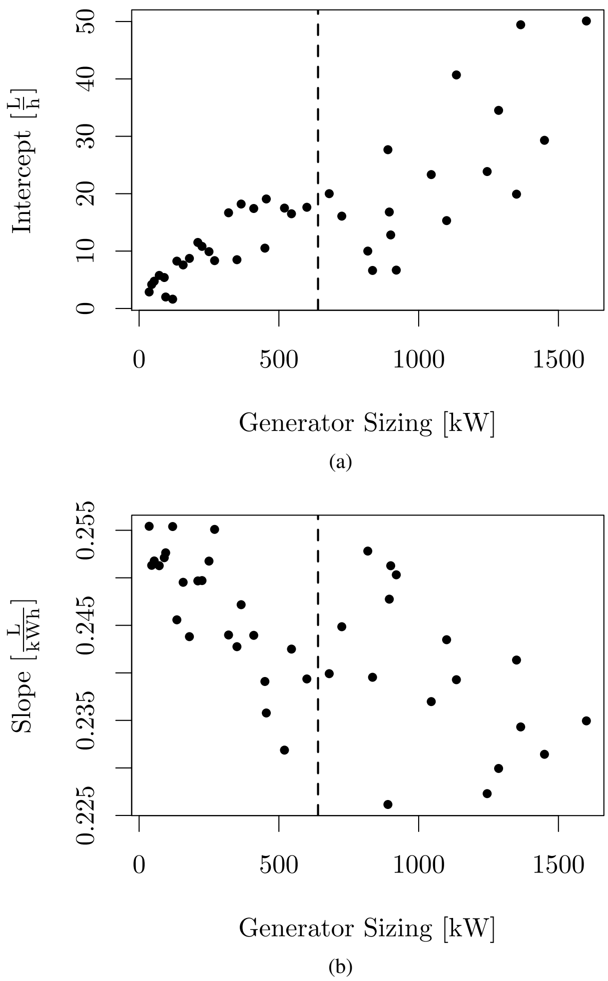


Fig. 4: Multilevel regression estimates of fuel consumption (a) intercept and (b) slope across generator sizing. The dotted vertical lines emphasize an apparent "break" in the data.

range of the linear coefficients estimated in this study, the importance of the slope and intercept with respect to the heat rate "flips". That is, accurate estimates of one linear coefficient are no more important than another across a wider range of sizings. However this does imply the accuracy of the intercept parameter is more important for smaller sized gensets, whereas the accuracy of the slope parameter is more important for larger sized gensets.

IV. General Fuel Consumption Model

A. Fitting the general model

Consider Fig. 4a and Fig. 4b, which show the fuel consumption slope and intercept estimates against genset sizing. Note that despite the heterogeneity of the variance being accounted for in (6), the variability of the linear coefficients appears to be non-constant with respect to genset sizing. Upon closer inspection however, the fit for both intercept and slope may be comprised of two distinct linear relationships, partitioned by the dotted vertical lines. This possibility is interesting in that it suggests some characteristic that differentiates genset fuel consumption about the 600 kW sizing. However caution must be used before ascribing too much importance to this feature of the data. The purpose of this study is to fit a general model of fuel consumption parametrized by genset sizing. There is convincing evidence of a broadly negative and positive relationship with the slope and intercept respectively across genset sizing. Accounting for the seeming break in the fit about 600 kW would be a case of over-fitting within this study's scope, especially lacking any satisfactory explanation for the pattern beyond what is seen in the presented data.

Linear fits estimated through ordinary least squares are employed to model the fuel consumption slope and intercept across genset sizings,

$$\text{slope}: \; b_2 \sim \mathcal{N}(\beta_0 + \beta_1 \mathrm{G}, \sigma^2_{\text{slope}}) \tag{9}$$

$$\text{intercept}: \; b_1 \sim \mathcal{N}(\beta_0 + \beta_1 \mathrm{G}, \sigma^2_{\text{int}}) \tag{10}$$

such that G represents genset sizing, with b_1 and b_2 the linear coefficients for heat rate in (1). Using the results from these fits, a general model for fuel consumption as a function of genset sizing is,

$$F = (\beta_{0\text{int}} + \beta_{1\text{int}} G) + (\beta_{0\text{slope}} + \beta_{1\text{slope}} G) P \tag{11}$$

$$\left.\begin{aligned} \beta_{0\text{int}} &= 3.63719 \\ \beta_{1\text{int}} &= 0.02031 \end{aligned}\right\} b_1$$

$$\left.\begin{aligned} \beta_{0\text{slope}} &= 0.25098 \\ \beta_{1\text{slope}} &= -1.1827 \times 10^{-5} \end{aligned}\right\} b_2$$

where P is a corresponding set of loadings $P \in [0.1 \cdot G, G]$. Recall that all datasheets used within this study provided fuel consumption data to a minimum of 10% and a maximum of 100% of rated power output.

B. Validity of the general model

Despite the potential violation of homogenous variance and linearity in the slope and intercept models, the normality assumption upon the error specified in (9) and (10) is useful to assess in validating the standard error of the regression coefficients. The Shapiro-Wilks (S-W) procedure is a popular and powerful test of univariate normality [17]. Using this test upon the residuals of (9) and (10) results in a failure to reject the normality assumption. Standard errors are given for the coefficients of (9) and (10) along with the p-values from the S-W test on both model's residuals in Table 2.

The potential of overfitting in (11) is investigated through a Monte Carlo cross validation. The slope and intercept estimates are randomly partitioned into 8 subsets of size 5. Of the 8 subsets, 7 are used as training data to refit (9) and (10). The remaining subset is used as validation data. The mean absolute error is used to evaluate the difference between the trained models and validation data. This process is repeated with 10,000 replications, representing approximately 1.5% of the possible permutations. The results in Fig. 5 show a consistent spread about a mean error of 6 $\frac{\mathrm{L}}{\mathrm{h}}$.

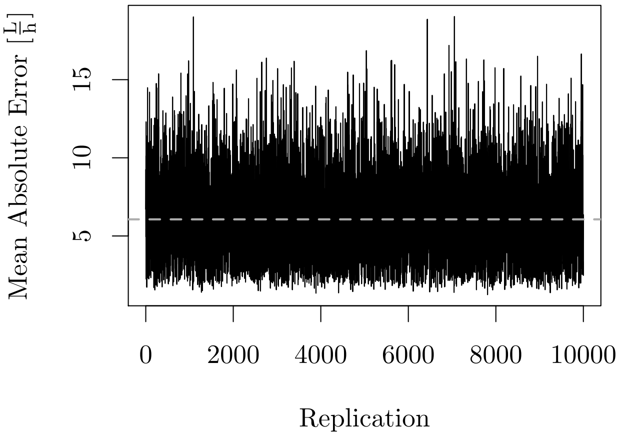


Fig. 5: Mean absolute error of Monte Carlo cross validation on (11).

TABLE II: Statistical parameters of (9) and (10)

	Model (9)		Model (10)	
	Slope	Intercept	Slope	Intercept
Standard Error	2.5×10^{-3}	1.9	2.1×10^{-6}	1.6×10^{-3}
P-Value	1.7×10^{-6}	≈ 0	7.6×10^{-10}	0.06
S-W P-Value	0.33		0.79	

C. Applying the general model

The non-linear transformation from fuel consumption to heat rate curve shown in (2) is applied to (11). To provide an illustrative assessment of the general model, new manufacturer data from a 925 kW prime diesel genset in [5] and experimental data from a 190 kW, 320 kW, and 457 kW prime diesel genset in [3] are compared to the general model's estimated curve. Fig. 6 shows the comparison, with the general model fit given as a solid line, the fresh data being the dashed line, and 99% confidence intervals given by the dotted lines. Note the confidence interval estimates are supported by S-W test's failure to reject the normality assumption for (9) and (10). The confidence intervals represent a range that comprises 99% of the general model's fits across hypothetical indefinite samples from the entire population of prime diesel generators within the prescribed loading range of genset sizings in this study. The general model is observed to have close approximations of the experimental data in (a) and (c), and less accurate approximations of the experimental data in (b) and the manufacturer data in (d). Both curves for the fresh data fall outside the 99% confidence interval in (b) and (d), however the approximation is still reasonably accurate. Note the abnormal behaviour in the experimental data in (b), the authors in [3] theorize the drop in efficiency could have been due to complications after a firmware update on the generator.

V. CONCLUSIONS

The general model of prime diesel generator fuel consumption parametrized by generator sizing has promise as a simple and convenient method to obtain fuel curves and subsequent heat rates for diesel gensets. Among the selection of gensets within this study, a relationship is identified between the slopes and intercepts of fuel consumption curves with respect to genset sizing. This was expected as genset sizing has an obvious relationship to genset fuel consumption. However, to fit the general model an explicit functional relationship between the fuel consumption linear parameters and genset sizing is needed. A multilevel regression process is compared to a more simplistic non-pooled regression to generate estimates for the slope and intercept of the fuel curves with respect to genset sizing. The multilevel regression proves superior through less overfit estimates. The multilevel regression's estimates of slope and intercept are shown to have a complementary role in affecting the heat rate.

A negative and positive linear trend is fit to the estimates of the fuel consumption curve's slope and intercept against genset sizing. There is a possibility of a more complex linear structure underlying the relationship between the fuel consumption slope and intercept and genset sizing, however in the interest of model parsimony only an overall linear trend was modelled. The linear fits of slope and intercept are used to parametrize a general model for fuel consumption. This general model is investigated through a Monte Carlo cross validation, which shows no serious concerns for model overfit.

The general model's ability to approximate new data is investigated. A combination of experimental and manufacturer diesel generator data is compared to the general model's curve. The results are generally acceptable, with 2 of 4 fits being very good approximations, fitting within the 99% confidence bounds.

The model estimated within this study is useful as a general estimate for a prime diesel generator's fuel consumption at a particular sizing. Simulations or modelling of diesel-integrated systems may be better served using this study's estimates of fuel consumption slope and intercept with respect to sizing to approximate fuel consumption instead of ad-hoc methods. The results from this research may be further studied and contrasted with additional validation data sources, or a similar modelling approach may be applied to a wider or deeper population of diesel generator sizings.

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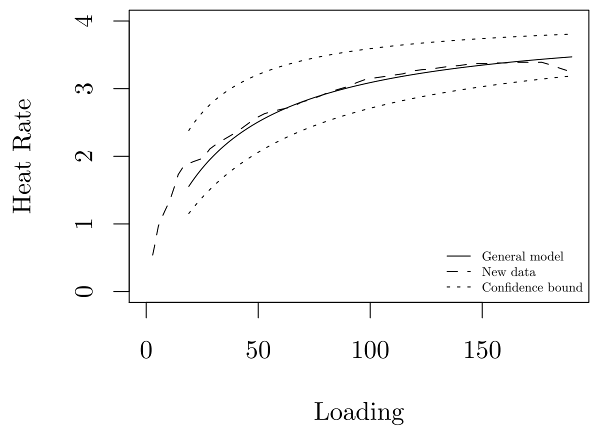

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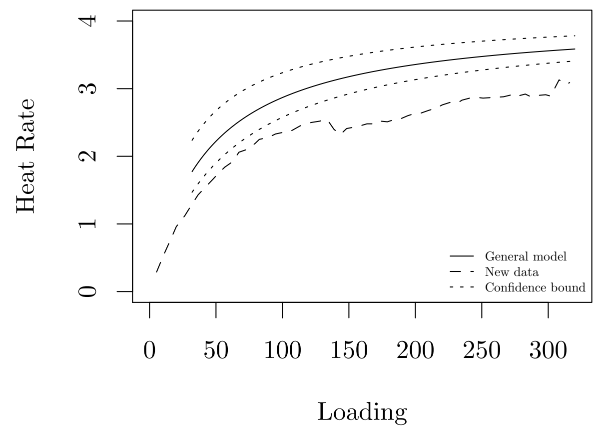

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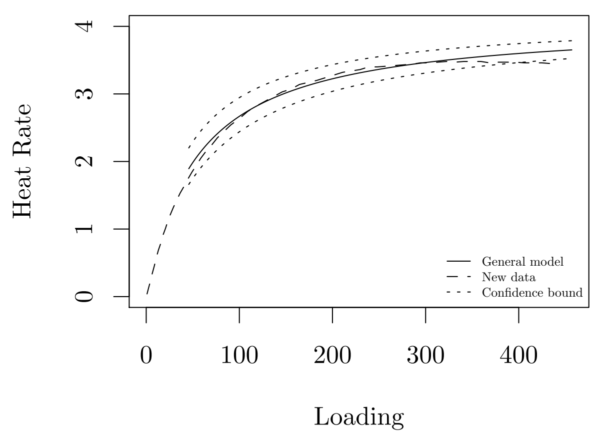

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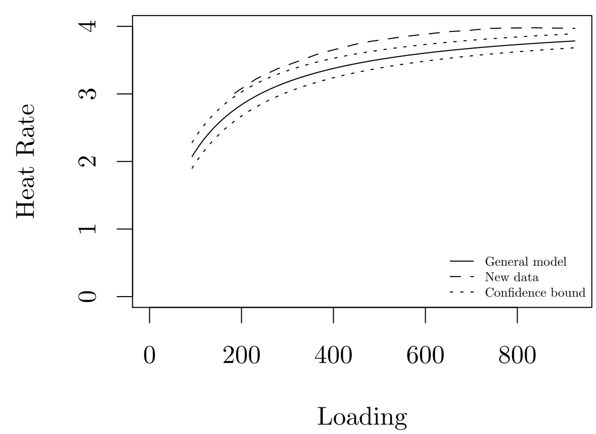

(d)

Fig. 6: Comparisons of new genset data to general fuel consumption model results. (a) compares a 190 kW genset, (b) compares a 320 kW genset, (c) compares a 457 kW genset, and (d) compares a 925 kW genset. The model has very close approximations for (a) and (c), and less close but still reasonable approximations for (b) and (d).

REFERENCES

[1] L. L. J. Mahon, *Diesel Generator Handbook*. Butterworth-Heinemann, 1992, ch. Load considerations, pp. 137–138.

[2] K. Tobalt, C. Shao, and M. Ross, “Diesel generator optimal dispatch controller,” Yukon University - Northern Energy Innovation, Tech. Rep., 2018.

[3] M. Mueller-Stoffles, J. VanderMeer, H. Schaede, L. Miranda, D. Light, A. Simmons, and H. Merkel, “Diesel generator fuel consumption under dynamic loading,” Alaska Center for Energy and Power, Tech. Rep., 2017.

[4] M. Ismail, M. Moghavvemi, and T. Mahlia, “Techno-economic analysis of an optimized photovoltaic and diesel generator hybrid power system for remote houses in a tropical climate,” *Energy Conversion and Management*, vol. 69, pp. 163 – 173, 2013.

[5] F. Katiraei and C. Abbey, “Diesel plant sizing and performance analysis of a remote wind-diesel microgrid,” in *2007 IEEE Power Engineering Society General Meeting*, 2007, pp. 1–8.

[6] W. Cai, X. Li, A. Maleki, F. Pourfayaz, M. A. Rosen, M. Alhuyi Nazari, and D. T. Bui, “Optimal sizing and location based on economic parameters for an off-grid application of a hybrid system with photovoltaic, battery and diesel technology,” *Energy*, vol. 201, pp. 1 – 23, 2020.

[7] Bagen and R. Billinton, “Evaluation of different operating strategies in small stand-alone power systems,” *IEEE Transactions on Energy Conversion*, vol. 20, no. 3, pp. 654–660, 2005.

[8] C. Segaser, “Conventional alternating-current generators and engine generator sets,” Oak Ridge National Laboratory, Tech. Rep., Apr. 1978.

[9] H. Energy, “Homer pro,” 2020. [Online]. Available: https://www.homerenergy.com/products/pro/docs/latest

[10] L. L. J. Mahon, *Diesel Generator Handbook*. Butterworth-Heinemann, 1992, ch. Prime mover and generator protection, pp. 382–383.

[11] A. Gelman and J. Hill, *Data Analysis Using Regression and Multilevel/Hierarchical Models*, 1st ed. Cambridge University Press, 2007, ch. Multilevel linear models: the basics.

[12] J. Pinheiro and D. Bates, *Mixed-Effects Models in S and S-PLUS*, 1st ed. Springer, 2000, ch. Extending the Basic Linear Mixed-Effects Model.

[13] “R: A language and environment for statistical computing,” R Core Team, 2019. [Online]. Available: https://www.R-project.org/

[14] J. Pinheiro, D. Bates, S. DebRoy, and D. Sarkar, “nlme: Linear and nonlinear mixed effects models (3.1-140),” 2019. [Online]. Available: https://CRAN.R-project.org/package=nlme

[15] J. Hox, M. Moerbeek, and R. Schoot, *Multilevel Analysis*, 3rd ed. Routledge, 2017, ch. Estimation and Hypothesis Testing in Multilevel Regression.
[16] D. Harville, “Maximum likelihood approaches to variance component estimation and to related problems,” *J. Am. Stat. Assoc.*, vol. 72, no. 358, pp. 320–338, 1977.
[17] H. Thode, *Testing For Normality*. Marcel Dekker, 2002.